MODELLING ON-FARM DIVERSIFICATION THROUGH PORTFOLIO OPTIMIZATION AND GOAL PROGRAMMING: A CASE STUDY FROM BOLIVIA

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ABSTRACT

Modern Portfolio Theory provides a theoretical framework for agricultural risk reduction. Powerful yet accessible tools have been developed to optimize scarce capital/labor allocation to increase returns and reduce correlated risks via diversification. Such tools are used to assess rural livelihood diversification induced by an incentive-based program for watershed conservation piloted between 2003 and 2011 in a context of rural poverty in Bolivia. The tools assembled and tested in this study may provide low-cost diagnostics to improve implementers' understanding of risks and returns in a specific rural context. Comparing alternative portfolio frontiers may represent a useful and transformative tool to understand socio-ecological systems such as watersheds, facilitating regime shifts that benefit ecosystem services and livelihoods.

Key-words: Socio-Ecological Resilience, Modern Portfolio Theory, Goal Programming.

INTRODUCTION

Agricultural risk reduction is an elusive goal and an urgent one for the global rural poor. Agricultural risk is site specific and multilayered, but similarly to risk in stock market investments, it can be reduced via adequate diversification. Modern Portfolio Theory and the set of tools available to compute alternative investment allocations represent a largely untapped resource to alleviate poverty by reducing agricultural risk. If agricultural risk reduction in itself does not dramatically increase incomes, at least it can stabilize them and potentially create room for savings and borrowing capability.

The case study for the application of portfolio theory to rural household economies was provided by a regional incentive-based watershed conservation program called *WaterShared* (Asquith 2016), piloted by Natura Bolivia Foundation, and adopted by a growing number of sub-national and national governments in Bolivia, Ecuador, Peru and Colombia. The *WaterShared* program has been ranked as a good practice for adaptive management (IIED 2010). Incentivizing alternative livelihoods is a recommended practice to neutralize leakage in conservation

projects where farmers enroll marginal land but continue deforesting on more suitable land (Angelsen, A. (ed.) 2008). Natura provides in-kind incentives and training to diversify household economy, productive assets and skills aimed at neutralizing leackage and alleviate poverty for households that enroll forest land in a conservation program. Beekeeping, fruticulture and intensified livestock management, in this order, have been promoted by Natura contributing to diversifying livelihoods away from the dominant productive matrix in the region: itinerant annual agriculture and extensive cattle rearing. Was introduced diversification adequate to reduce risks and increase returns?

Poshiwa et al. (2013) used asset portfolio metrics to assess how conservation initiatives may help reducing rural households´ annual income fluctuations due to rainfall variation through diversification of wildlife use in Zimbabwe, relating agricultural risk reduction to alternative livelihoods in favor of conservation. Similar studies tend to estimate agricultural returns based on farm-gate price historical data and a "one size fits all" costs estimates based on secondary information (Seitz and Torre 2014). Though a reasonable approximation with potential to generate sound investment recommendations for farmers, the outcome of such studies may oversimplify the real-world complexity given by heterogeneity across farmers. There is a wild variability of returns in a single agricultural season and across a relatively homogeneous region: timing in all important to i.e. get the right amount of rain at the right time, or a good price at harvest. Skills and assets vary widely across a sample of farmers as well as random fluctuations in local markets and climates.

The rural context across the developing world is one of informal economy, poverty and accounting illiteracy. The cost of collecting field data at the household level is high and few institutions ever generate proper time series. Recent trends in program evaluation are considering "right-fit" evidence strategies for program monitoring and improvement (). Thus, the objectives of this paper are to (i) demonstrate the use of cross-sectional field data to determine expected returns from, and variability of, various land-based activities; and (ii) to calculate the optimal capital and labor allocation following the principles of modern portfolio theory (Markowitz, 1952; 1959) adopting rather inexpensive cross-sectional data. Presented here is the result of a quest for a robust yet "right-fit" method to qualify introduced on-farm diversification.

The resilience framework and modern portfolio theory.

A unifying paradigm emerged in the past three decades to frame adaptive co-management of social systems and the ecological systems they depend upon. That unifying and still unfolding paradigm is called Socio-Ecological Resilience (Holling 1986, 2001; Folke et al. 2002, 2016). Social and ecological resilience are related and increasingly treated as features of unified socio-ecological systems (SES), which are nested systems with complex feedbacks and interactions, subject to environmental changes due to natural and human induced processes. Resilience is defined as the ability to maintain functions under stress and bounce back from

shocks (Walker and Salt 2006). Resilience can be described as the capacity to respond to "unforeseen surprises" without losing the system's integrity and functionality and is chiefly maintained by diversity (Walker and Salt 2012; Nemec et al. 2013).

How can external experts recommend or assess if diversification is the "right" kind? For instance: 1) diversifying crops that have the same agricultural calendar (planting and harvest time) may actually worsen risk rather than reduce it. 2) If expected returns on investments (ROI) are high on average, but extremely variable , the asset may not be appropriate for a stable portfolio. Assets such as capital, labor, productive inputs and skills will be understood as synonymous to the productive activities they enable. To cross the language bridge with finance we may also say that rural livelihoods allow just long-only positions. I intend to qualify the asset and households' diversified portfolios in terms of risk reduction. Markowitz (1952; 1959) devised enduring principles for capital allocation meant to diversify away investments' risk while maximizing returns: modern portfolio theory (MPT). The model for portfolios of risky assets is conceptually simple (minimize the variance (x_i ; ... x_h) and co-variance (σ_{ih}) of expected ROI while maximizing ROI (r_i ; ... x_i), formally expressed as:

$$Min \; \frac{1}{2}\sum_{i,h=1}^{n} x_i \; x_h \; \sigma_{ih}; \qquad\qquad Max \sum_{i,h=1}^{n} r_i \; x_i$$

Subject to:
$$\sum_{i=1}^{n} x_i = 1; \qquad x_i \geq 0; \qquad i = 1 \dots n; \qquad x \in D$$

Diversification *per se* is no panacea in a setting of limited capital it may prevent specialization with negative impacts on farmers income (Townsend 1994; Morduch 1995). As Markovitz (1952) originally put it: "Since the future is not known with certainty, it must be "expected" returns which we discount [...] If we ignore market imperfections the foregoing rule *never* implies that there is a diversified portfolio which is preferable to all non-diversified portfolios." Markovitz's procedure for portfolio selection is a mathematically sound best guess to reduce exposure to non-systematic risk based on the admittedly unverified assumption that the future will look just like the past.

The maximum entropy principle (MEP) has been successfully used in diverse fields ranging from population ecology to quantitative finance (Harte, 2011; Usta and Kantar, 2011) in order to extend prior knowledge by finding the least biased distribution consistent with that knowledge. Entropy is a measure of diversity and is at its maximum when portfolio weights are equally distributed. Informational entropy represents the number of bits required to encapsulate the information relative to a probability distribution. An equally distributed four assets portfolio is an example of maximum entropy, portfolio's entropy is at its maximum and equals 0.60 bit. Entropy may be expressed as a percentage of this maximum value (which varies according to the assets included in a portfolio). One empirical study on

financial time series found that the solution to the optimization problem that maximizes entropy (H) of the assets' weights, while keeping risk low, performed better in out of sample (OOS) tests than the classical solution that maximizes returns keeping risk low (Usta and Kantar, 2011). Less efficient, more diversified and sub-optimal asset selection, performed better than optimal solutions OOS (Rongxi et al. 2013; Geman et al. 2014). Applying the MEP reduced the risk of over-fitting the model to the data and allows for more performing portfolio selection facing the fundamental unpredictability of the real world. Through goal programming is it possible to test assumptions in a flexible framework of optimal farming investment allocations including an H target function:

$$\text{Information Entropy } (H) = -\sum_{i=l}^{k} pi \ln pi$$

Where *pi* represents the share of capital allocated to an asset included in the portfolio. Applying the MEP in this context equals to setting a higher diversity target in addition to reducing risk and increasing expected returns, potentially generating better performing portfolios out of sample. If diversity is one of the main ingredients in the recipe for adaptive performance in both theories 1) socio-ecological resilience and 2) modern portfolio, and it is measurable as entropy on available cross-sectional data, then we can qualify rural households' portfolios consistently for the time period under exam (12 months).

MATERIAL AND METHOD
The dataset

The original dataset is a cross-section with valid surveys from 97 households, based on 2015-2016 returns on investment in the main on-farm assets. Assets were aggregated in four categories: 1) annual agriculture, 2) fruticulture, 3) livestock and 4) beekeeping. Such dataset represents a detailed reconstruction of on-farm cash flow by activity for a sample that can be classified almost homogeneously as poor and very poor, territorially spread on five Municipalities. The impacts on income of local micro-climatic conditions, markets, skills, capital, etc. and the *WaterShared* program are all discounted by a single metric: return on investment in each relevant asset during the 12 months prior to the survey. Households' production costs are represented mainly by own-labor, monetized at an average rate of USD 14.60 for agricultural daily wages in the study area. Return on investments were computed as follows:

- Cost: monetized own labor based on local daily wages + inputs and services acquired.
- Net income by crop/asset: ((own-consumption + production sold) * average price) - Cost.
- Return on investment (ROI): Net income / Cost.

Theory and Calculations

Data augmentation and portfolio optimization

Portfolio optimization algorithms begin with the calculation of the variance-covariance matrix. Most households surveyed rely on two or three main on-farm assets, so the dataset built with ROI per asset presents empty cells (sheet 1 in attached spreadsheet). Quantitative is a field plagued with partial and incomplete time series. The solution developed and adopted in the field consists of a commonly used data augmentation technique that allows generating synthetic data (correlated normals) with the same statistical properties as the original dataset. Portfolio level sensitivity and value-at-risk (VAR) analyses are rarely done other than via historical simulation (i.e. Andersen, G. et al., 2007). Parametric methods are used extensively to develop historical simulations and reduce to a minimum the likelihood of "unforeseen surprises", exploring the correlated risks of low-probability/high-impact events in the furthest section of the PDF´s tail.

The calculation of the augmented matrix starts with a symmetrical covariance matrix calculated on the original dataset (a *real* co-variance matrix). Missing values determine pairwise exclusion. Eigenvalues and eigenvectors are calculated on this covariance matrix, computed on the original data. The transformation matrix is the result of matrix multiplication of transposed eigenvectors times the square root of the diagonal matrix of eigenvalues. Correlated normal are obtained multiplying the uncorrelated random normals array by the transformation matrix.

The standard procedure for portfolio optimization is then applied to computed standard normals whose averages, standard deviations (STD), correlation and covariance matrixes are identical to those calculated on the field data (sheet 2 in attached spreadsheet). (Abbot, K. 2013). The algorithm used to nonlinearly optimize portfolios according to alternate goals is the generalized reduced gradient (GRG) developed by Lasdon, L. et al. (1973), calculations were implemented in excel 2016 (Wright, C. 2012 a, b).

RESULTS AND DISCUSSION

Rural poverty and agricultural risk

Classifying surveyed beneficiaries with the Unsatisfied Basic Needs Index 89% of 109 can be defined as poor and very poor in terms of assets and economic capability by national standards (INE, 2004). The best performing asset provided by the program for this relatively homogeneous group was beekeeping, averaging a ROI above 400% which obviously doesn't include beehives acquired or received as an incentive before the current agricultural year. Such excellent performance is due to a very low investment required to maintain the beehives, most surveyed farmers barely invest in renewing pre-printed wax or provide bees with additional calories in winter (amounting to USD 3-4/year). The bulk of the cost in 2015-2016 was represented by own labor invested in harvesting honey twice a year, averaging 13 work-days. Noticeably most surveyed farmers (60%) depend primarily on annual agriculture and secondarily on livestock, worst performing assets on both ROI and risk.

Table 1. Returns and risks associated to each asset in 2015-2016.

	Annual Agric. ANAG	Livestock LIVE	Fruticulture FRUT	Beekeeping BEE
Expected Returns (ROI)	104%	135%	374%	407%
Standard Deviation (STD)	434%	350%	1263%	727%
Coefficient of Variation (CV)	417%	258%	338%	178%

Efficient portfolios

The sector's benchmark for the risk-free investment is the agricultural daily wage, averaging USD 14.60 in the study area. The most efficient diversified portfolio (2) did not select livestock but only beekeeping and fruticulture, while the second-best portfolio (3: Maximize Slope Benchmark / CV constraint: Entropy target 40%) selected 2% of total capital/labor available within the household to be invested in livestock. Higher H targets (4, 5) increase the share of capital allocated to livestock and fruticulture, turning such selections into better candidates for OOS performance. A qualitative leap appears when risk is conceptualized in absolute as opposed to relative terms (6) with a drop in ROI, sliding abruptly on the inefficient side of the portfolio frontier. Risk conceptualization in relative terms generates higher returns but smallholder farmers will rarely take paper-risk. The most risk averse allocation choice (min STD) corresponds to local preferences as the majority of surveyed households primarily relies on annual agriculture and covets more livestock. In this Bolivian case-study a mix of expert opinion and local preferences helped selecting adequate on-farm diversification (as per 2015-2016 field data) through a participatory process. The incentivized productive assets (beekeeping, fruit trees and intensified livestock) and diversified portfolios including combinations of them generate some of the most efficient portfolios whit higher H targets (slightly sub-optimal). Despite persisting asset poverty and relatively low agricultural incomes it is clear that Watershared supported alternative livelihoods and contributed to transition household economies up the efficient side of the portfolio frontier by increasing capital/labor allocation to Beekeeping, fruticulture and intensified livestock management.

Table 2. Portfolio selection, rank ordered by decreasing ROI.

| N.	Target Function	Assets					Performance / statistics			
		ANAG	FRUT	LIVE	BEE	TOT	ROI	STD	CV	H
1	Maximize Expected Returns (ROI)	0%	0%	0%	100%	100%	407%	823%	202%	0%
2	Maximize Slope: Benchmark / CV	0%	8%	0%	92%	100%	405%	806%	199%	19%
3	Entropy target 40%	0%	17%	2%	81%	100%	396%	783%	198%	40%
4	Entropy target 60%	2%	20%	8%	71%	100%	375%	727%	194%	60%
5	Entropy target 80%	6%	22%	16%	57%	100%	340%	642%	189%	80%
6	Maximize Slope: Benchmark / STD	0%	14%	45%	41%	100%	279%	449%	161%	72%
7	Maximize Diversity	25%	25%	25%	25%	100%	255%	498%	195%	100%
8	Minimize Coefficient of Variation	0%	15%	68%	17%	100%	217%	319%	147%	61%
9	Minimize Standard Deviation	25%	6%	65%	4%	100%	153%	275%	180%	67%
10	Minimize Expected Returns	100%	0%	0%	0%	100%	104%	442%	424%	0%
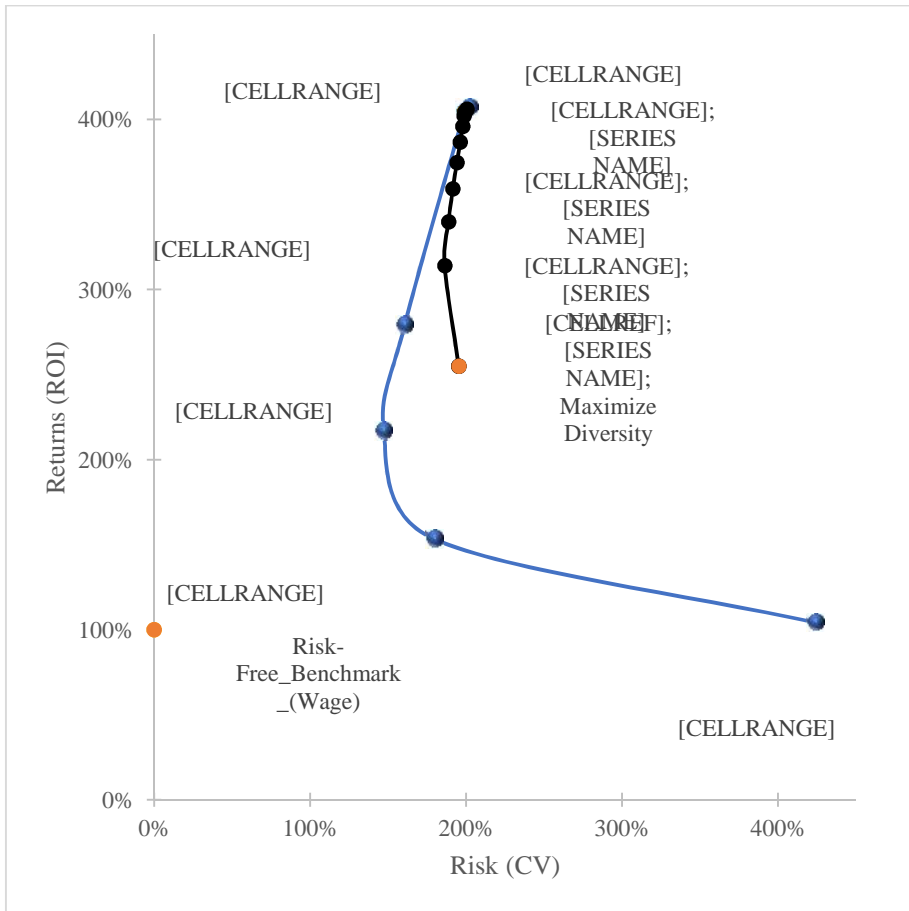
Graph 1. Portfolio Selection by Target Function.

These results corroborate the conclusion arrived at by others that useful in-kind incentives and access to new markets, as well as "intangible" benefits, such as training, may explain participation in conservation schemes better than opportunity cost analyses (Kosoya, et al. 2007; IIED 2007; Muradian et al. 2009; Bottazzi et al. 2018). Agricultural risk reduction provides a motivation to engage in a program such as watershed conservation and has merit in and of itself for household wellbeing.

One interesting result is that the relationship between ROI and H is curvilinear, best described by a polynomic curve of 2nd order ($R^2=0.81$, see graph 2), in confirmation of the fact that misdirected diversification may *worsen* the exposure of smallholder farmers with detrimental impacts on income of smallholder farmers. As allocations improve, expected returns increase and risk is reduced, increased diversity of assets correlates positively with ROI and negatively with risk. Increase in diversity is associated with increase in allocation efficiency only on the inefficient side of the curve.

Once the turning point to the efficient side of the curve is passed (RI = 255% for "maximum diversity") and the allocations improve in favor of ROI, diversity decreases with a negative correlation. Efficiency improvements increase both risks and return, showing how diversity per se is not the target of actions aimed at risk reduction. Agencies involved in agricultural risk reduction as an incentive to protect environmental services or as a goal in itself, can make use of portfolio optimization and apply it to relatively inexpensive cross-section data.
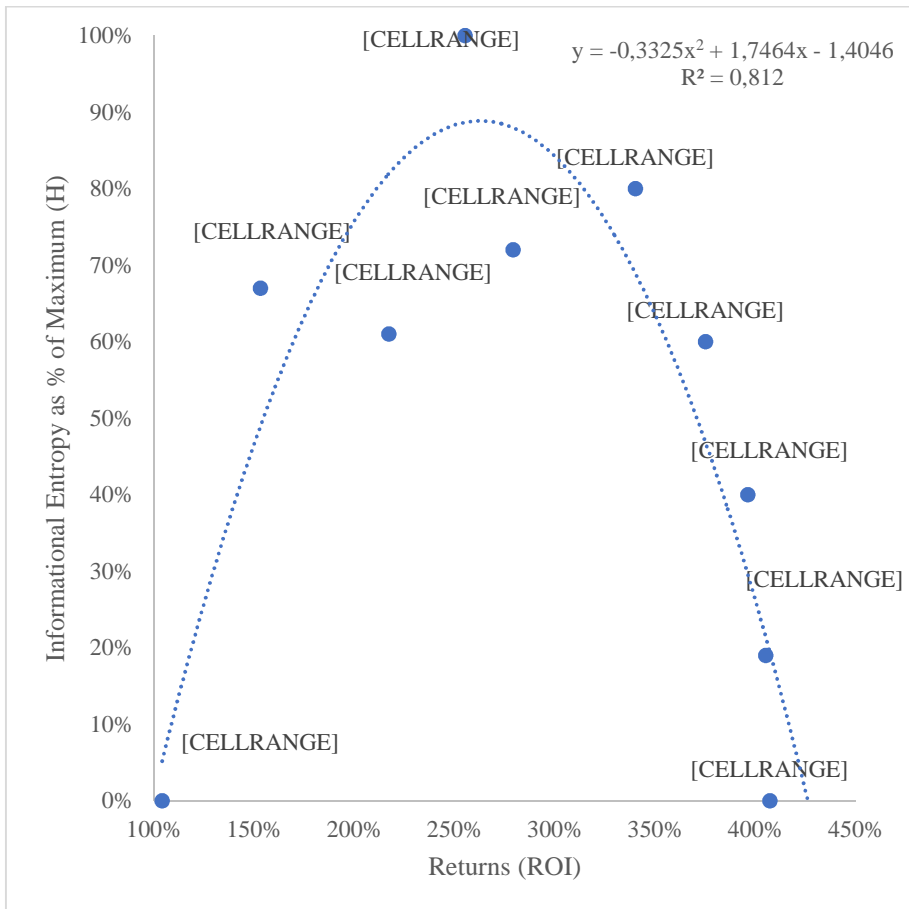
Graph 2. relationship between ROI and H is curvilinear.

CONCLUSIONS

In this and similar rural contexts misdirected diversification may have harmful impacts on both the environment and livelihoods, i.e. the most risk-averse allocation (min STD) relies heavily on annual agriculture and livestock, representing a local minimum trap and being associated with deforestation and forest degradation. Risk-aversion and constraints shape households´ preferences but not necessarily in their best interest. Public and private services may assess livelihoods in the target population to fine tune extension services.

While beekeeping is often a relatively new business for beneficiary households, orchards are known to generate returns without the annual investments (and risks) implied by annual agriculture. The entry barrier to get started with this business is given by the fact that the establishment of orchards requires at least three years until the first fruits can be harvested, an unaffordable use of land for most asset poor households. Natura's program incentivizes planting fruit trees on degraded land, providing the kick-start incentive to break annual agriculture long fallow cycle. The combination of local knowledge and preferences with environmentally friendly development options generated optimal choices to maximize social, economic and environmental goals at the same time. These tools can support and accelerate desired transformative change in a socio-ecological system, helping farmers and agricultural extensionists to make more informed investment decisions.

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